Movie Probability

ABOUT THIS LESSON

This lesson is designed to enhance students’ understanding of simple and compound events and their complements using information presented in Venn diagrams and a two-way table. From these two displays of data, the lesson provides practice in determining probability and also introduces conditional probability.

OBJECTIVES

Students will
- interpret organized data.
- compute probabilities of simple and compound events.
- compute conditional probabilities.

LEVEL

Grade 7 in a unit on probability

MODULE/CONNECTION TO AP*

Probability

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MODALITY

NMSI emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using those representations indicated by the darkened points of the star to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.

P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical
COMMON CORE STATE STANDARDS FOR
MATHEMATICAL CONTENT

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill.

Targeted Standards
7.SP.8a: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
(a) Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
See questions 1c, 1e, 2c-f, 3f-h

Reinforced/Applied Standards
7.SP.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
See questions 1b-e, 2, 3f-h

6.SP.5a: Summarize numerical data sets in relation to their context, such as by:
(a) Reporting the number of observations.
See questions 1a, 3a-e

COMMON CORE STATE STANDARDS FOR
MATHEMATICAL PRACTICE

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. NMSI incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

MP.1: Make sense of problems and persevere in solving them.
Students decipher the information in the Venn diagram and two-way table and select the appropriate quantities to determine the probability of specific events.
FOUNDATIONAL SKILLS
The following skills lay the foundation for concepts included in this lesson:

- Calculate simple probabilities
- Subtract a fraction between 0 and 1 from 1

ASSESSMENTS
The following formative assessment is embedded in this lesson:

- Students engage in independent practice.

The following additional assessments are located on our website:

- Probability – 7th Grade Free Response Questions
- Probability – 7th Grade Multiple Choice Questions

MATERIALS AND RESOURCES

- Student Activity pages
TEACHING SUGGESTIONS

Teachers should note that students are asked to read both non-overlapping and overlapping Venn diagrams. A non-overlapping Venn diagram represents mutually exclusive events, which are events that cannot occur at the same time.

The complement of an event is defined as the probability that an event does not occur and is calculated as one minus the probability that the event does occur, 1 – P(event does occur). The complement of set A can be written $A^c$. This notation is sometimes read: “not A” and $P(A^c) = 1 - P(A)$.

In question 1d, the concept of a “complement” is informally introduced while question 1e uses the word “complement” to essentially ask the same question. In these questions, the complement is one minus the probability that a student saw either movie, $1 - \frac{37}{50} = \frac{13}{50}$.

Computing probabilities from a two-way table involves using the frequencies listed in the table. Consider the following scenario: Students on a field trip to the natural history museum were assigned to two groups. Each group was given a different color wrist band. The information is presented in the two-way table.

<table>
<thead>
<tr>
<th></th>
<th>Red</th>
<th>Green</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>27</td>
<td>28</td>
<td>55</td>
</tr>
<tr>
<td>Girls</td>
<td>23</td>
<td>22</td>
<td>45</td>
</tr>
<tr>
<td>Total</td>
<td>50</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

- The 100 in the last cell represents the total number of students on the trip.
- The cell containing 55 represents the total number of boys on the trip.
- The cell containing 27 indicates that 27 students had a red wristband and were boys.

Using this information, the probability of randomly selecting a student who has a red wristband and is a boy would be $P(\text{Red and Boy}) = \frac{27}{100} = 0.27$.

In this lesson, conditional probability is introduced in questions 2d, 2e, and 3g. The probability is calculated based on a given condition and limits the sample space. For example, if the question asks for the probability of choosing a student with a red wristband given that the student is a boy, the phrase “given that” limits the sample space for the probability to boys, in this case, the first row of the two-way table. The notation for the conditional probability is $P(\text{Red | Boy}) = \frac{27}{55} \approx 0.491$, in other words, since we know the student is a boy, the sample space for the question is 55 boys, and the boys with red wristbands represent the favorable outcomes. Another way to ask this conditional probability question could be, “Of the boy students in the class, what is the probability of choosing a student with a red wristband?” Once again, the sample space is limited to just boys.

To extend this lesson, ask the students to select a topic, survey their classmates, organize the data, and then write probability questions based on their data.

You may wish to support this activity with TI-Nspire™ technology. See Working with Fractions and Decimals in the NMSI TI-Nspire Skill Builders.

Suggested modifications for additional scaffolding include the following:

1e Provide the process used to calculate the complement, $1 - \text{probability the event occurs} = \text{complement}$.

3f-h Modify each question to ask “how many…” first. For example, modify question 3f to ask: “How many students saw both Movie C and Movie D? What is the probability that a student selected at random from the 7th grade saw Movie C and Movie D?”
**NMSI CONTENT PROGRESSION CHART**

In the spirit of NMSI’s goal to connect mathematics across grade levels, a Content Progression Chart for each module demonstrates how specific skills build and develop from sixth grade through pre-calculus in an accelerated program that enables students to take college-level courses in high school, using a faster pace to compress content. In this sequence, Grades 6, 7, 8, and Algebra 1 are compacted into three courses. Grade 6 includes all of the Grade 6 content and some of the content from Grade 7, Grade 7 contains the remainder of the Grade 7 content and some of the content from Grade 8, and Algebra 1 includes the remainder of the content from Grade 8 and all of the Algebra 1 content.

The complete Content Progression Chart for this module is provided on our website and at the beginning of the training manual. This portion of the chart illustrates how the skills included in this particular lesson develop as students advance through this accelerated course sequence.

<table>
<thead>
<tr>
<th>6th Grade Skills/Objectives</th>
<th>7th Grade Skills/Objectives</th>
<th>Algebra 1 Skills/Objectives</th>
<th>Geometry Skills/Objectives</th>
<th>Algebra 2 Skills/Objectives</th>
<th>Pre-Calculus Skills/Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compute the probability of simple events and their complements.</td>
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<tr>
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<td>Compute the probability of compound events and their complements.</td>
<td>Compute the probability of compound events and their complements using the general addition and multiplication rules.</td>
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</tr>
</tbody>
</table>
Movie Probability

Answers
A probability answer that has not been reduced to simplest fractional form is acceptable and may help the teacher to quickly determine if the student has an understanding of the concept. If desired, students can simplify the fractions or convert to decimal form for extra practice and reinforcement of basic numeric skills.

1. a. 13
   b. \( \frac{20}{50} \)
   c. \( \frac{37}{50} \)
   d. \( \frac{13}{50} \)
   e. \( \frac{13}{50} \)

2. a. \( \frac{37}{50} \)
   b. \( \frac{20}{50} \)
   c. \( \frac{14}{50} \)
   d. \( \frac{9}{17} \)
   e. \( \frac{8}{32} \)
   f. \( \frac{15}{37} \)

3. a. 55
   b. 5
   c. 35
   d. 27
   e. 28
   f. \( \frac{7}{60} \)
   g. \( \frac{7}{35} \)
   h. \( \frac{20}{25} \)
Movie Probability

1. This data was collected and recorded from both 6th grade classes at our school. All the students in both classes were asked if they saw Movie A or B. The Venn diagram displays the results.

![Venn diagram with 20 in Movie A circle and 17 in Movie B circle]

a. If there are a total of 50 students in both classes, how many students saw neither of the two movies? Write the answer in the Venn diagram inside the rectangle, but outside the two circles.

b. What is the probability that a student selected at random from the 6th grade saw Movie A?

c. What is the probability that a student selected at random from the 6th grade saw either Movie A or Movie B?

d. What is the probability that a student selected at random from the 6th grade saw neither of the two movies?

e. What is the complement of the probability that a student selected at random from the 6th grade saw either Movie A or Movie B?

2. There are 50 students in the two 6th grade classes.

<table>
<thead>
<tr>
<th>Gender</th>
<th>Movie A</th>
<th>Movie B</th>
<th>Did not see either movie</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>14</td>
<td>8</td>
<td>10</td>
<td>32</td>
</tr>
<tr>
<td>Female</td>
<td>6</td>
<td>9</td>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
<td>17</td>
<td>13</td>
<td>50</td>
</tr>
</tbody>
</table>

a. What is the probability that a student selected at random from the 6th grade saw one of the movies?

b. What is the probability that a student selected at random from the 6th grade saw Movie A?

c. What is the probability that a student selected at random from the 6th grade saw Movie A and was a male?

d. Of the students who saw Movie B, what is the probability that the student was a female?
e. What is the probability that a student selected at random from the 6th grade saw Movie B given that the student was male?

f. Given that a student saw either Movie A or Movie B, what is the probability that a student selected at random was female?

3. In two 7th grade classes at our school, 60 students were asked if they saw Movie C, Movie D, or both Movie C and D. The Venn diagram displays the results.

![Venn diagram](image)

a. How many students saw at least one of the movies?

b. How many students saw neither of the movies?

c. How many students saw Movie C?

d. How many students saw Movie D?

e. How many students saw only Movie C?

f. What is the probability that a student selected at random from the 7th grade saw Movie C and Movie D?

g. Of the students who saw Movie C, what is the probability that a student saw both movies?

h. Of the students who did not see Movie C, what is the probability that they saw Movie D?